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FORMALIZING DHARMAKĪRTI

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Introduction

There are at least three ways of formalizing a system of logic that is not expressed in a formal or artificial language (such as one of the systems of modern logic), but in a natural language (such as Sanskrit, Chinese or Tibetan). The first may be called the *logical* method. It is:

> not to state the Indian authors' words into symbols, but express what they meant to say – i.e., what they would have said if they had used precisely the same language as we do (R.S.Y. Chi 1969: 15).

Harbsmeier (1998: 371) quotes this passage and characterizes it by its use of *mathematical logic* without raising the question to what system is referred to by the phrase "precisely the same language as we do."

I take it that Chi means by "language": "artificial language" and by "we": "modern logicians." Although there is no single logic that can be called "modern," I am not averse to use the phrase. I would use it in the same vague and general sense that we use the phrase "modern science" about which Joseph Needham has written:

> To write the history of science we have to take modern science as our yardstick – that is the only thing we can do – but modern science will change, and the end is not yet (Needham 1976: xxix).

I accept *modern logic* as our yardstick. But before we apply it we must make sure first what the authors we study *say*.

Harbsmeier uses a different method to which he refers as *philo-logical*. He distinguishes his method from "proto-logical adumbrations of modern logical concepts." Instead, it offers:

> A conceptual scheme in its own right. A conceptual scheme not to be reduced to modern logic, but to be more fully understood by comparison with modern theories (Harbsmeier *ibid.*).

In the present article I will use a third method which I shall call *linguistic*. I regard it as preparatory to the other two. The reason is that it stays as close as possible to the original form of the terms, especially the technical terms, of the sample, large or small, of logic we are studying and seeking to clarify. It turns out to be presupposed by both the *logical* method, which involves a certain amount of translation of that original form, and the *philo-logical*, which involves not only translations but introduces one or more conceptual schemes.

Before I gave it a name, I practised the linguistic method in my early studies of Indian Nyāya and Navya-nyāya (Staal 1960, 1961, 1962a, reprinted in 1988). But strangely enough, I did not use that method in dealing with Dharmakīrti (Staal 1962b, also reprinted in 1988). I propose to clarify the matter and in so doing illustrate what I mean by *linguistic*. Provisionally I would describe it as follows:

The *linguistic* approach to the study of a logical expression formulated in a natural language, such as Sanskrit, Chinese or Tibetan, is to look at the *form* of the expressions.

The chief part of my paper deals with some famous expressions from Dharmakīrti using the particle *eva*. In the end I shall comment, more briefly, on the even more celebrated *catuṣkoṭi* due to Nāgārjuna and developed in his school about which there exists already an enormous literature.

I. *eva* एव

My example from Dharmakīrti occurs in the *Nyāyabindu* (ed. Stcherbatsky 1918, 1970: 18-9; Candrasekhara Sastri 1954: 22-3; Malvania 1955, 1971: II.5):

(1) *liṅgasyānumeye sattvam eva* लिङ्स्यानुमेये सत्त्वमेव

(2) *sapakṣa eva sattvam* सपक्ष एव सत्त्वम्

(3) *asapakṣe cāsattvam eva* असपक्षे चासत्त्वमेव

Before discussing the formalization of these logical expressions, one or two facts about the notion of "formal" and the philosophical background should be mentioned. Indian logicians never created a fully formal or artificial

language such as Pāṇini and other Indic grammarians had done (Staal 1995, 2006). But they were concerned, as many philosophers were, about being *exact* and *precise*. Many of their expressions may be described as "formal" in that more general sense. It applies in particular to expositions of the two valid means of knowledge which all Indic philosophies agreed in accepting: perception (*pratyakṣa*) and inference (*anumāna*). Within the Buddhist tradition, it was already known to Vasubandha and Asaṅga that inference should satisfy the three conditions that correspond to (1) – (3). That was not an invention of Dharmakīrti, nor was it due to his great predecessor Dignāga who formulated eight conditions and wrote his treatise on the valid means of knowledge (*Pramāṇasamuccaya*; cf. Hattori 1968: 4 and *passim*).

Dharmakīrti's emphasis on *sattvam* "existence" and his reference to his own work as *sattvānumāna*, "inference based on existence," supplements the inference based on non-existence that had been used by other Buddhist thinkers to argue for momentariness. The important point is this. All parties agreed that it was necessary to adopt the same notion of logical inference, which should be expressed in as exact and precise terms as could be found. Only a universally accepted logic would enable philosophers to effectively argue with and refute each other. Such cases are well attested in the history of Indic philosophies. In the most striking cases, the defeated opponent accepted the philosophy of the victor by whom he had been defeated in a public debate.

Almost half a century ago, in 1962, I formalized Dharmakīrti's expressions, using a mixture of linguistic and logical methods. I wrote that I formalized the threesome in a manner "which preserves as far as possible the structure

of the Sanskrit expressions"; and I introduced, for that reason, the *restricted variables* I had used before in the study of Nyāya expressions and that are related to the *lambda* introduced into modern logic by Alonzo Church. I was successful to some extent, because I gave the reader who is familiar with modern logic an idea of the linguistic structure of these expressions. But I was not consistent because what I presented was at the same time a logical argument that was intended to show that Dharmakīrti expressions (2) and (3) were contrapositives. That was simply due to the fact that the topic of my paper was *Contraposition*. My sequences of definitions and expressions appear to have established, that Dharmakīrti did not realize that he was dealing with contraposition though it was clear that his commentator Dharmottara did. That difference was not perceived by Stcherbastky, Bocheński or Frauwallner. The scholar who had come closest to seeing it was Randle (1930: 183).

The method I used myself was more logical than linguistic but it failed to fully "preserve the structure of the Sanskrit expressions" because it did not fully analyze the Sanskrit particle *eva* एव and its position in Dharmakīrti's logic. That was unfortunate because that particular use of *eva* was, in all probability, his chief contribution to logic.

According to traditional interpretations, *eva* expresses emphasis or stress: *avadhāraṇa*. It is placed after the expression which it emphasizes and excludes everything else, e.g.:

(4) *prajāpatir ha vā idam agra eka evāsa*

प्रजापति ह वा इदमग्र एक एवास

"Prajāpati was here in the beginning all by himself" (*Śatapatha Brāhmaṇa*);

(5) *tvam eva yantā nānyo 'sti pṛthivyām*

त्वमेव यन्ता नान्योऽस्ति पृथिव्याम्

"you only are the charioteer, no one else on earth" (*Mahābhārata*);

(6) *aham eva kariṣyāmi*

अहमेव करिष्यामि

"I myself will do it" (Pañcatantra).

Dharmakīrti himself in his *Pramāṇavārttikā Svopajñavṛtti* gives two examples from ordinary Sanskrit of the use of *eva*. They raise several problems which have been discussed by Kajiyama (1973:162 = 1989:156) and Hayes and Gillon (1991: 31-2; cf.) but need not concern us here.

The particle *eva* may be translated into English as "only," "just," "precisely," "exclusively," etc., or by special phrases such as "all by himself," "myself" or "very." It generally applies to *preceding* phrases. They need not precede *immediately*, like *prajāpati* in (4) which is separated from *eka eva* by four other words.

In our three examples, the preceding or special phrases are, respectively: *prajāpati*, *tvam* ("you") and *aham* ("I"). All of these are nouns or Noun

Phrases (NP). These are either nouns or larger expressions that contain nouns and function as nouns. The Noun Phrases "I myself" functions like the pronoun "I" and the name "Prajāpati" might be replaced by a Noun Phrase such as "ruler of the universe." The particle *eva* may also occur after a Verb Phrase (VP), under similar conditions, not exemplified here.

If I were to try to provide a general description in linguistic terms of *eva* in these cases and in many others, I would say, it is used after a Noun Phrase (NP) or Verb Phrase (VP):

(7) NP/VP-*eva*

and means in English;

(8) "only NP/VP" or "only NP/VP and nothing else."

We must now formalize Dharmakīrti's expressions (1) – (3) adopting the linguistic approach. First of all, we need some letter variables to refer to the basic terminology. The term *liṅga* or "sign" refers to the *hetu* or "reason" which I shall refer to as *h*. It is explicitly mentioned only in (1), but understood in (2) and (3). The term *anumeya* or "thing-to-be-inferred" is more often called *sādhya* and I shall refer to is as *s*. The term *pakṣa* is basically simple although it has been made almost unintelligible (Staal 1973). It may be translated as "instance" or "locus" but we should avoid "subject" which introduces an Aristotelian category that is extremely misleading in the study of Indic logics. I refer to it a *p*, and to *sapakṣa*, "similar instance" or "similar locus," as *p**. *Sattvam* means "existence" and I

shall express it as a relational predicate because it always presupposes a locus: *A(h, s)* or "*h* exists in *s*." I shall use "~" as a symbol for negation. We are now left with *eva* which I shall call **μ**. (1) – (3) may now be reformulated as:

(1*) A(*h, s*) **μ**
(2*) A(*h, p** **μ**)
(3*) ~ A(*h,* ~*p**) **μ**.

In (4) – (7), *eva* occurs after a NP or Noun Phrase. The reader may check once again that it does. Note that *sattvam* "existence" is a NP but "exists" (as in the English below) is not. We are now in a position to make use of (8) which explains what *eva* means in English and enables us to arrive at customary translations such as:

(1**) "the reason exists in the locus";
(2**) "the reason exists in all similar instances";
(3**) "the reason does not exist in any dissimilar instances."

What have we gained by this? We have used **μ formalization** to illustrate the genius of Dharmakīrti, who used *eva* to express what in modern logic would, in general, be expressed by quantification. Without formalization, it was put in clear terms, perhaps for the first time, by Yuiji Kajiyama : "Dharmakīrti demonstrates a kind of quantification theory of the affirmative proposition" (Kajiyama 1973:162 = 1989:156).

The only historian of logic who understood Dharmakīrti's logical use of *eva* was I.M. Bocheński (1956: 505-6). He made a mistake because he was not a linguist: he wrote that *eva* in Sanskrit applies to the subject or predicate of a sentence though it applies only to a Noun or Verb Phrase. Bocheński was not only a historian of Greek logic (see, e.g., Bocheński 1951), but the first to make Indian logic part of a general history of logic.(He did not include Chinese logic, which was included in a philosophical encyclopedia in 1967 by A.C.Graham, long before Graham 1978).

Bocheński was an excellent logician in his own right, a Dominican priest as indicated by the letters "O.P." (Tymieniecka 1965), Rector of the University of Fribourg in Switzerland, and passionate about fast cars. In that domain he made good use of logic: when his physician told him that he was too old to race. he used inference and changed his doctor.

II. *catuṣkoṭi* चतुष्कोटि

I now come to the second and final part of my talk which is more sketchy and dispenses with most references to editions and the scientific literature because there exists already a vast literature about it. It includes the magisterial survey by Ruegg (1977) which, after another quarter century of active research, is unavoidably dated. That does not affect that it provides good illustrations for the *linguistic method* which are moreover based, at least in part, on a more recent work: Tillemans 1999.

As for Nāgārjuna's own expressions, they are so well-known I need hardly quote them – but here they are:

sarvaṃ tathyaṃ na vā tathyaṃ	सर्वं तथ्यं न वा तथ्यं
tathyaṃ cātathyam eva ca	तथ्यं चातथ्यमेव च
naivātathyaṃ naiva tathyam	नैवातथ्यं नैव तथ्यम्
etad buddhānuśāsanam	एतद्बुद्धानुशासनम्

"All is just so, or not just so,
both just so and not just so,
neither just so nor not just so:
this is the graded teaching of the Buddhas"
(transl. Ruegg, *p.*6)

Nāgārjuna's disciple Āryadeva formulates the *catuṣkoṭi* as follows:

sad asat sadasac ceti	सदसत्सदसच्चेति
sadasan neti ca kramah	सदसन्नेति च क्रमः
eṣa prayojyo vidvadbhir	एष प्रयोज्यो विद्वद्भिर्
ekatvādiṣu nityaśaḥ	एकत्वादिषु नित्यशः

"Being, non-being, [both] being and non-being,
neither being [nor] non-being:
such is the method that the wise
should always use with regard to identity
and all other [theses]." (transl. Tillemans, *p.*189).

Tillemans begins his interpretation as follows: "Clearly, the conjunction of these four negations would make a Western logician dizzy, since, at first glance, the Mādhyamaka offers us the following four statements:

A. $\sim P$

$\sim\sim P$

$\sim(P \,\&\, \sim P)$

$\sim(\sim P \,\&\, \sim\sim P)$. "

I quote these four statements because they happen to illustrate almost precisely what I mean by *linguistic method*: they confine themselves to what the expressions show "at first glance." I say "almost" because what we read at first glance really is:

A.^{0} P

$\sim P$

$P \,\&\, \sim P$

$\sim(P \,\&\, \sim P)$.

After ten pages of discussion, Tillemans offers what he regards as "a translation into formal logic," that is, what would seem to be the result of applying the *logical* method:

B. $\sim(Ex)Fx$

$\sim(Ex)\sim Fx$

$\sim(Ex)(Fx \,\&\, \sim Fx)$

$\sim(Ex)(\sim Fx \,\&\, \sim\sim Fx)$.

Tillemans has greatly contributed to our understanding of Buddhist logic and it is not my purpose to discuss whether "B" is or is not a result of applying the logical method. I use it as an example of an interpretation in terms of concepts (such as the existential quantifier *E* and the concept of predicate or function *F*) that do *not* occur in "A." Unlike "A," it makes Nāgārjuna mean something that is very different from what he *says*.

To mean something different from what one says is not uncommon in philosophy but hardly acceptable in logic – *any* logic. It is part of the concept of "logic" that a logician tries to say as clearly as he can what he means. In logic, as in philosophy, clarity is not enough; but lack of clarity is fatal. If Nāgārjuna means something that is different from what he says, it may not detract from his status as a philosopher, but it affects the question whether he belongs to the Indic tradition of *logic*. As for Dharmakīrti, he tries to say as clearly as he can what he means to say. He is a logician in the constitutive meaning of the term; and in addition, original and brilliant.

There might be a way out for Nāgārjuna and his school. In order to explain it, let us look at the context of Tindemans' discussion which is entitled: "Is Buddhist Logic Non-Classical or Deviant?" I am not concerned with the difference between these two types of alternative logics, but all such logics possess a particular "field of application" from which they derive their justification (as Tindemans acknowledges: *p.* 191). If Nāgārjuna's alternative expressions had such a field of application, and/or his otherwise misleading expressions are helpful or useful in relation to that field of application, or even to other goals he had in mind and may have discussed

elsewhere, there might be a justification for his expressions. However, none of this is clear at least to me and neither is Nāgārjuna's "field of application."

An example of a clear "field of application" in an alternative logic occurs in *Intuitionist Logic*. It also requires complex expressions that are not at first sight clear and others of which it is not obvious why one is accepted , e.g.:

$$A \text{ implies } \sim\sim A \text{ ,}$$

and another one is not, e.g.:

$$\sim\sim A \text{ implies } A.$$

Intuitionist Logic is based upon the *Intuitionism* of the mathematician L.E.J. Brouwer and has been formalized by Brouwer's pupil A. Heyting. Its justification derives from the fact that Brouwer is concerned with questions concerning the infinite for which there may not exist, at present or at any given time, a satisfactory answer. A simple example is the value of the number **pi** which may be given as: 3,14159... . A better approximation is: 3,141592653589793238462... . Chinese and Indian mathematicians arrived at good approximations long before Europeans did. Computers have gone much further but they will never reach the end because the sequence of decimals is infinitely long.

Brouwer wondered whether we will ever know whether the mini-sequence 5555 will occur in that infinite sequence. We would know that the answer is

Yes if we had found such a sequence. But as long as we have not found it, we will not know what the answer is.

Let us see what happens when we substitute for A in the two above expressions: "5555 occurs in the sequence." We know that it is true that

$$A \textit{ implies } \sim\sim A$$

because "5555 occurs in the sequence" implies that it is not the case that 5555 has not been found.

Let us now assume that 5555 has not been found, i.e., $\sim A$. That does not imply that 5555 does not occur in the sequence because we might find that it does to-morrow. In other words, if it is not the case that 5555 has not been found, it does not follow that A or that $\sim A$. Hence it is not true that

$$\sim\sim A \textit{ implies } A.$$

Brouwer concluded that, in infinite sets, the principle of logic according to which "either A or $\sim A$" makes no sense. He rejected it accordingly, along with several other well known principles of classical, that is: pre-intuitionist logic and many theorems and other results of classical mathematics. He also introduced numerous new concepts, principles and theorems. Another example comes from the field of real numbers, where he introduced the relation:

$$(a \# b),$$

which means "*a lies apart from b,*" that is, there are some real numbers between a and b. That is different from the inequality relation:

$$(a \neq b),$$

which merely asserts that a is not the same number as b.

In classical mathematics, one implies the other. In intuitionism,

$$(a \# b) \text{ implies } (a \neq b)$$

but not:

$$(a \neq b) \text{ implies } (a \# b).$$

The concern about losing large chunks of classical mathematics prevented the large majority of working mathematicians from following Brouwer. His intuitionism is not, however, dead: it survives in different forms in *Recursive Functions* and *Constructive Mathematics* (see, e.g., Heyting 1959).

The point of mentioning these facts in our present context is that, in the case of Brouwer's intuitionism, there are numerous unexpected and perhaps uncontrollable implications; and there is a decidedly large "field of application": infinity. That applies to other "alternative logics" but does not, to the best of my knowledge, hold true of Nāgārjuna.

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